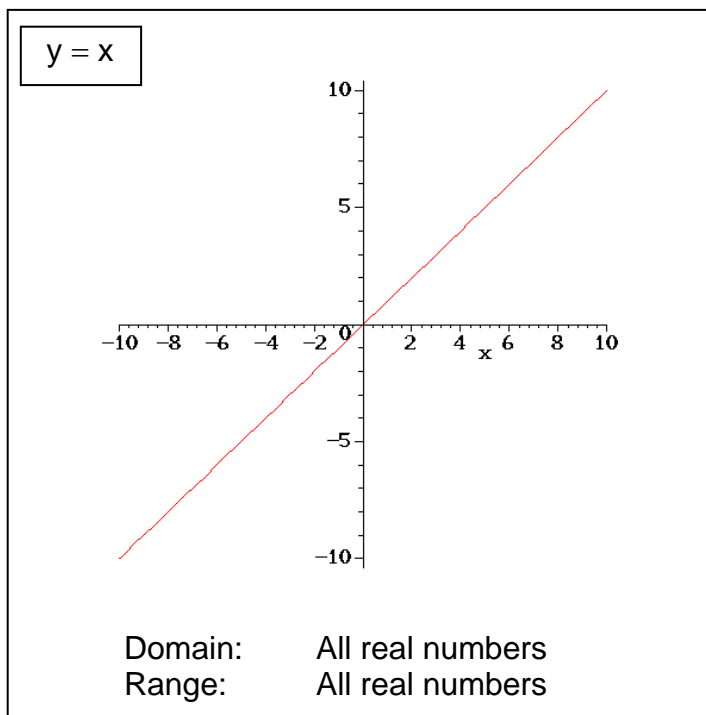Douglas College Learning Centre

EIGHT SPECIAL FUNCTIONS AND THEIR GRAPHS

To use this worksheet you should be comfortable with graphing functions and finding domain and range. If you aren't, speak with your tutor.

Knowing how a function behaves when it is graphed is an important advantage you can have when solving many math problems. This worksheet illustrates eight common functions that have unique graphs. As you work through it, try to focus on the patterns of each kind of function. Once you get used to what each kind of function looks like, you will hopefully be able to imagine the approximate graphs of these functions just by looking at the equations.

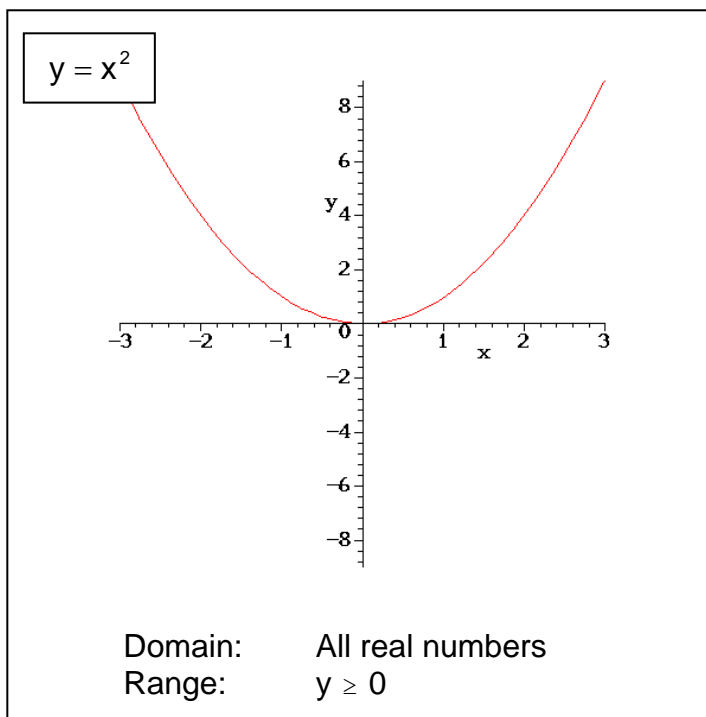The following table briefly defines each function in this handout:

Function	Definition	Example
Linear	The highest power over the x variable is 1	$y = 2x + 1$
Quadratic	The highest power over the x variable(s) is 2	$y = -2x^2 - 7x + 2$
Cubic	The highest power over the x variable(s) is 3	$y = x^3 + 2x^2 + x$
Square root	The x variable is square rooted	$y = -\sqrt{x} - 1$
Absolute value	The x variable is within absolute value signs	$y = -\lvert x + 1 \rvert$
Rational	The x variable is in the denominator ($x \neq 0$)	$y = \dfrac{3}{x}$
Logarithmic	The log of the x variable is taken	$y = \log(x)$
Exponential	The x variable is an exponent above a number	$y = a^x$

Linear functions

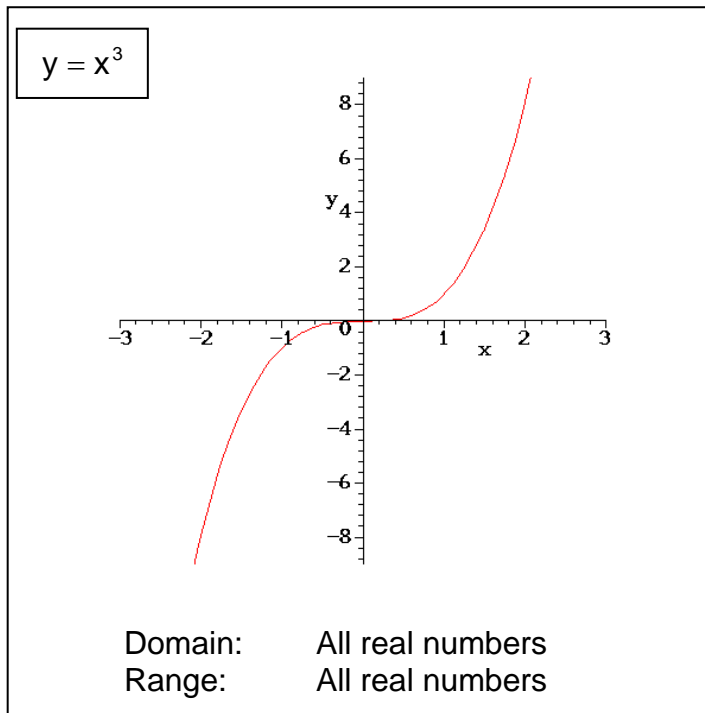$y = x$

Graph the following:

1a) $y = x + 1$

1b) $y = -2x - 1$

Domain: All real numbers
Range: All real numbers

Quadratic functions

$y = x^2$

Graph the following:

2a) $y = x^2 + 1$

2b) $y = 3x^2 - 1$

Domain: All real numbers
Range: $y \geq 0$

Cubic functions

$y = x^3$

Domain: All real numbers
Range: All real numbers

Graph the following:

3a) $y = x^3 - 1$

3b) $y = \dfrac{1}{2x^3} + 1$

Square root functions

$y = \sqrt{x}$
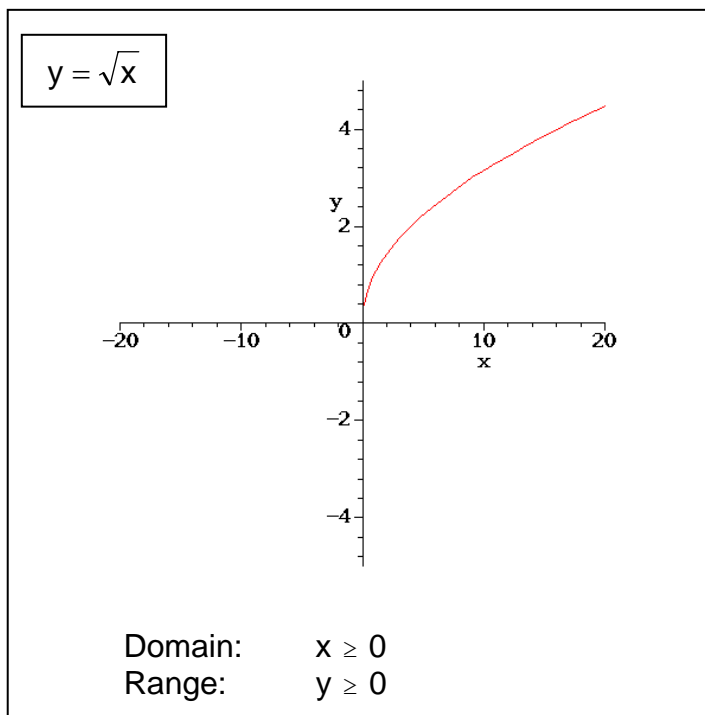
Domain: $x \geq 0$
Range: $y \geq 0$

Graph the following:

4a) $y = \sqrt{x + 1}$

4b) $y = \sqrt{x} + 1$

Absolute value functions

$$y = |x|$$
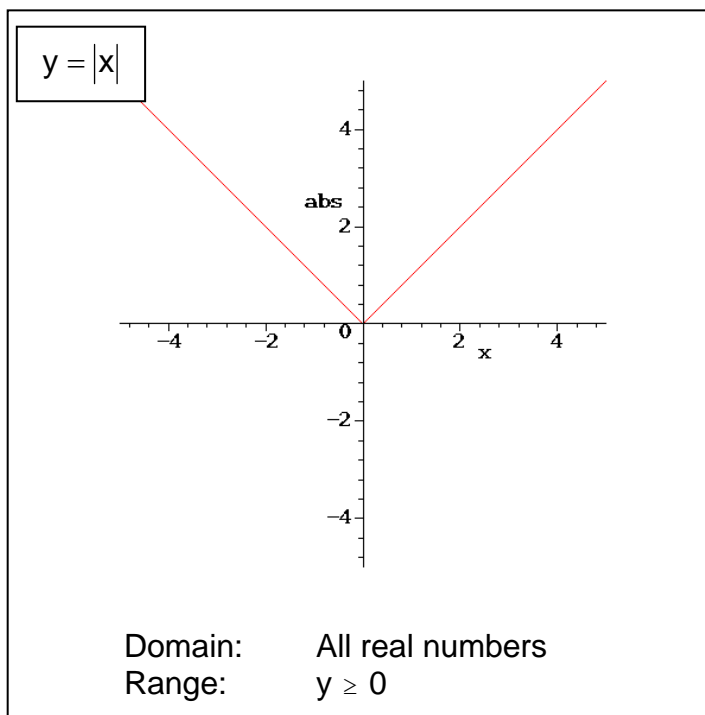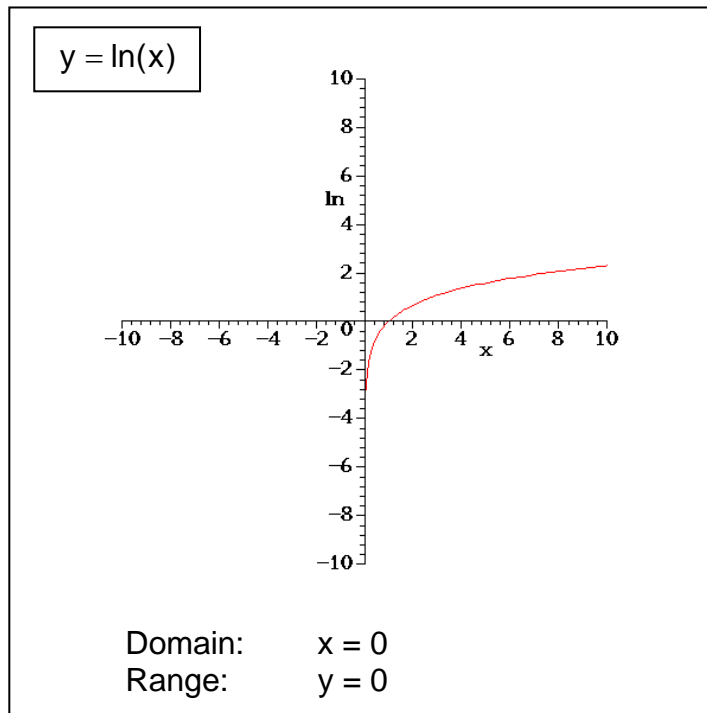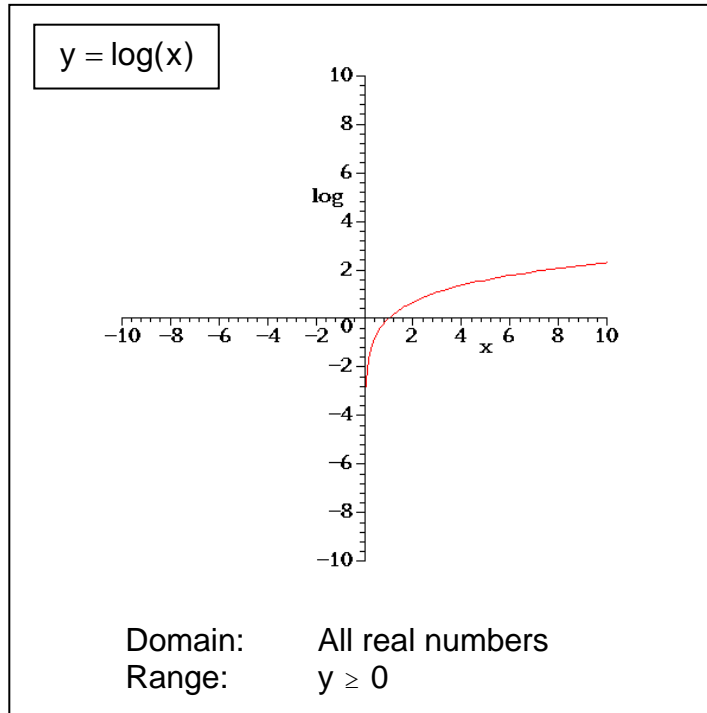

Graph the following:

5a) $y = |x| + 1$

5b) $y = |x + 1|$

Domain: All real numbers

Range: $y \geq 0$

Rational functions

$$y = \frac{1}{x}$$
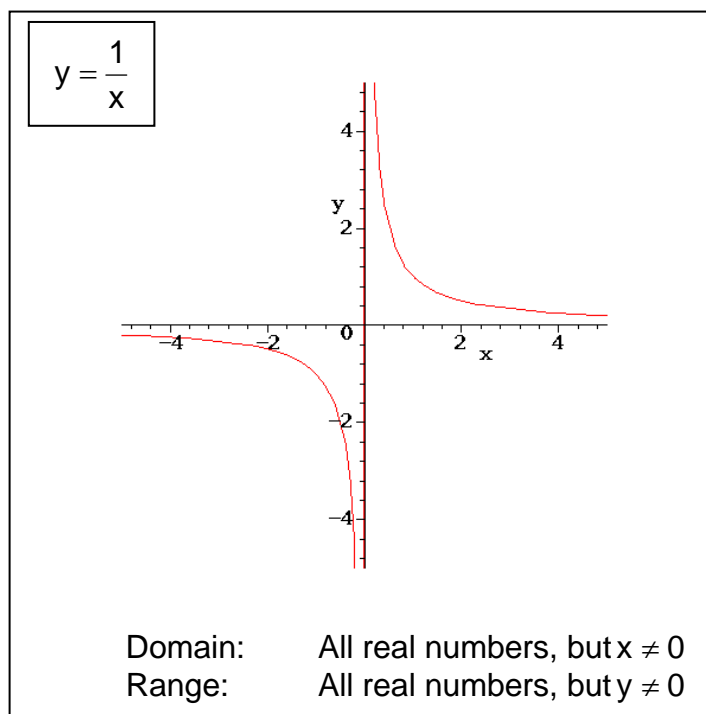
Graph the following:

6a) $y = \dfrac{1}{x+1}$

6b) $y = \dfrac{2}{x-3}$

Domain: All real numbers, but $x \neq 0$

Range: All real numbers, but $y \neq 0$

Logarithmic Functions

$y = \log(x)$

```
          10
           8
           6
  log
           4
           2
                        _____
-10 -8 -6 -4 -2  0   2  4   6  8  10
          -2        x
          -4
          -6
          -8
         -10
```

Domain: All real numbers
Range: $y \geq 0$

$y = \ln(x)$

```
          10
           8
           6
  ln
           4
           2
                        _____
-10 -8 -6 -4 -2  0   2  4   6  8  10
          -2        x
          -4
          -6
          -8
         -10
```

Domain: $x = 0$
Range: $y = 0$

Exponential functions

$y = a^x, a > 1$

Domain: All real numbers
Range: $y \geq 0$

$y = a^x, 0 < a < 1$
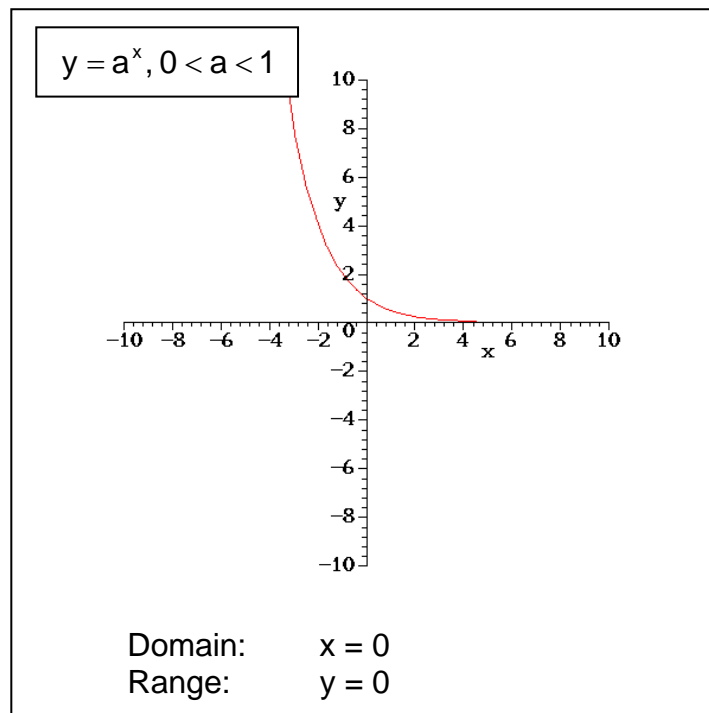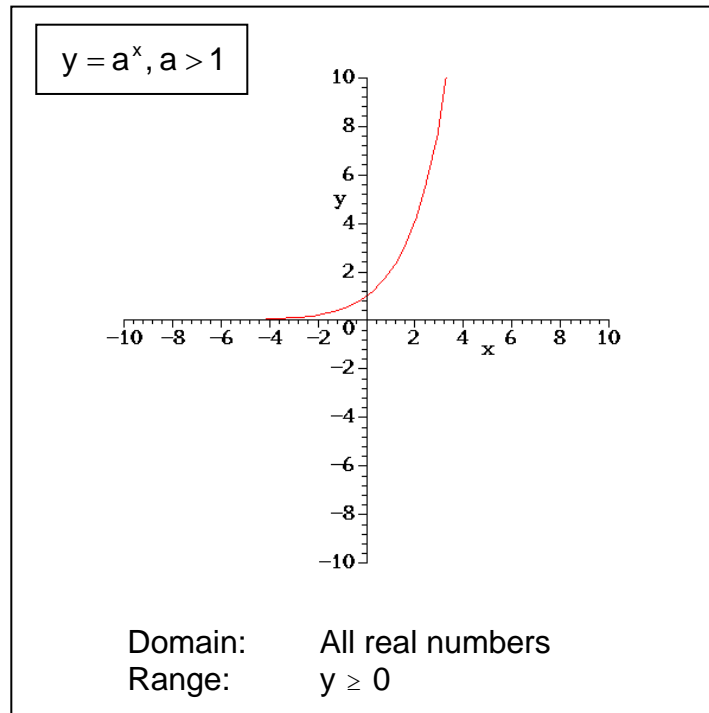
Domain: $x = 0$
Range: $y = 0$

Review Exercises:

Graph and state the domain and range for the following functions.

7) $y = -4x^2 + 5$

8) $y = \dfrac{-x^2}{2} - 3$

9) $y = -3x^3$

10) $y = \dfrac{-x^3}{2} + 5$

11) $y = -\sqrt{x}$

12) $y = \sqrt{x - 2}$

13) $y = -|x|$

14) $y = \dfrac{-1}{x + 2}$

15) $y = \dfrac{-3}{x - 6}$

Answers:

1a) $y = x + 1$
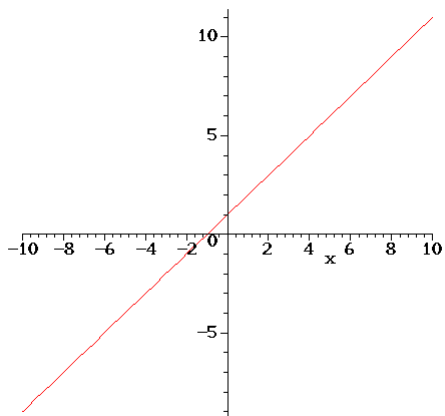
D = All real numbers
R = All real numbers

1b) $y = -2x - 1$
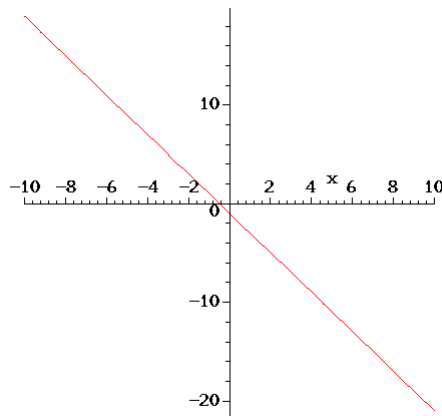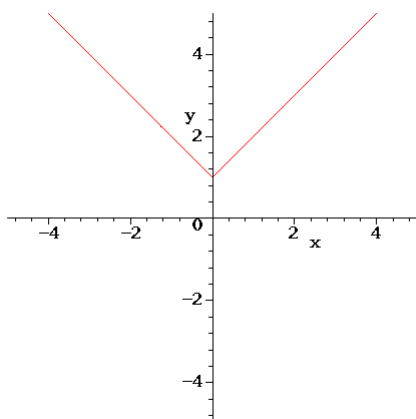

D = All real numbers
R = All real numbers

2a) $y = x^2 + 1$
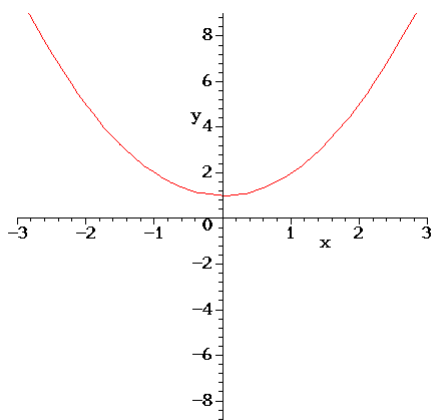
D = All real numbers
R = y ≥ 1

2b) $y = 3x^2 - 1$
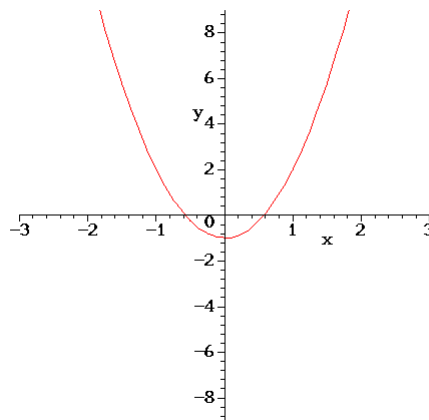
D = All real numbers
R = y ≥ -1

3a) y = x³ - 1
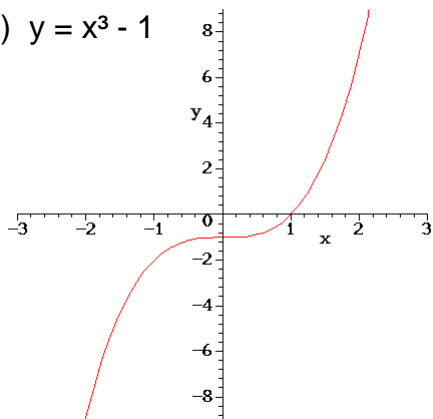
D = All real numbers
R = All real numbers

3b) $y = y = \dfrac{1}{2x^3} + 1$
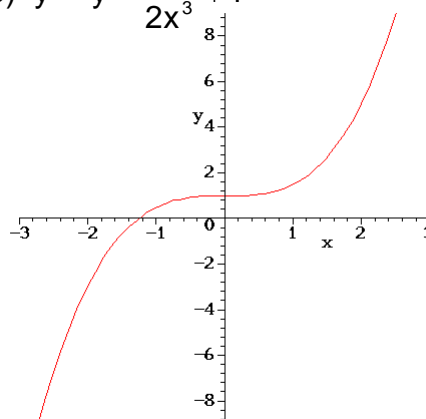
D = All real numbers
R = All real numbers

4a) $y = \sqrt{x+1}$
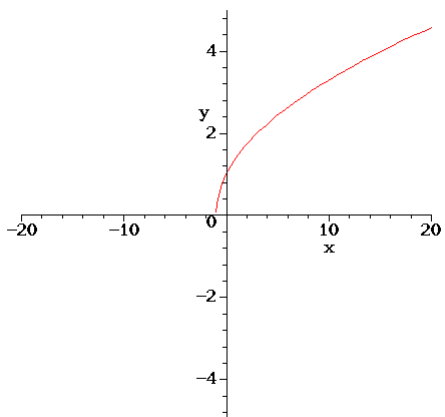
D = x ≥ -1
R = y ≥ 0

4b) $y = \sqrt{x} + 1$
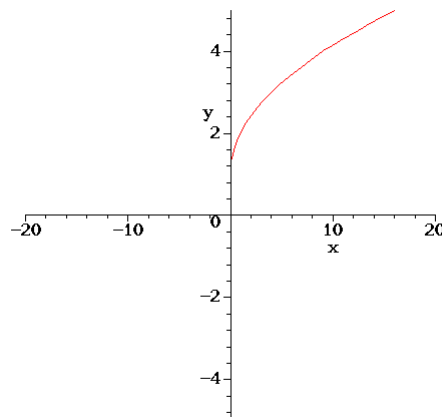
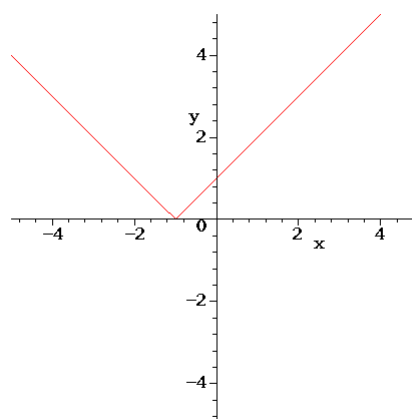D = x ≥ 0
R = y ≥ 1

5a) y = |x| + 1

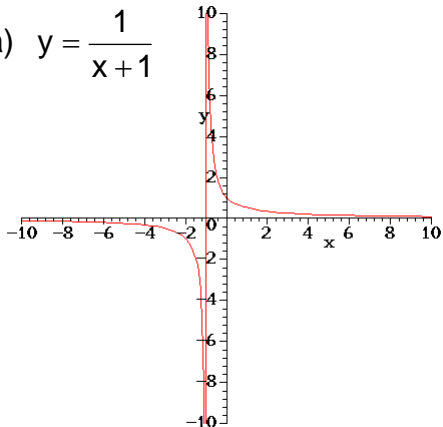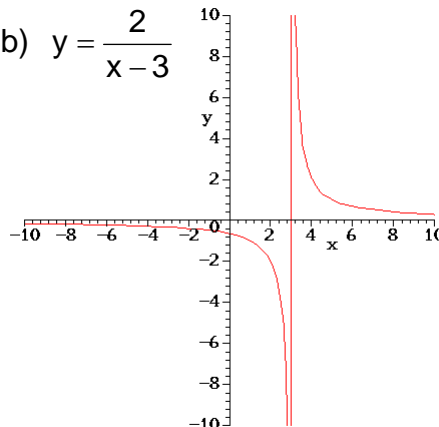D = All real numbers
R = y ≥ 1

5b) y = |x + 1|

D = All real numbers
R = y ≥ 0

6a) $y = \dfrac{1}{x+1}$

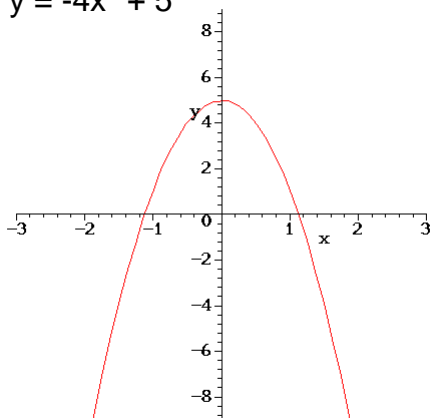D = All real numbers, but x = -1
R = All real numbers, but y = 0

6b) $y = \dfrac{2}{x-3}$

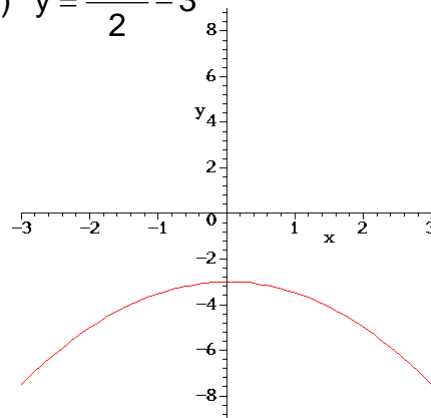D = All real numbers, but x = 3
R = All real numbers, but y = 0
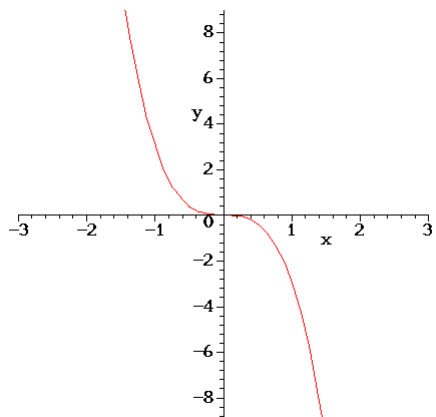
Answers to Review Exercises:

7) $y = -4x^2 + 5$

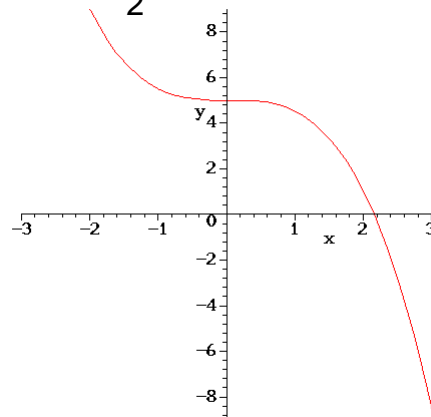D = All real numbers
R = y ≤ 5

8) $y = \dfrac{-x^2}{2} - 3$
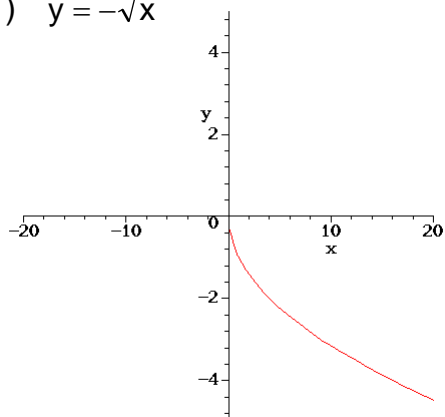

D = All real numbers
R = y ≤ -3

9) $y = -3x^3$

D = All real numbers
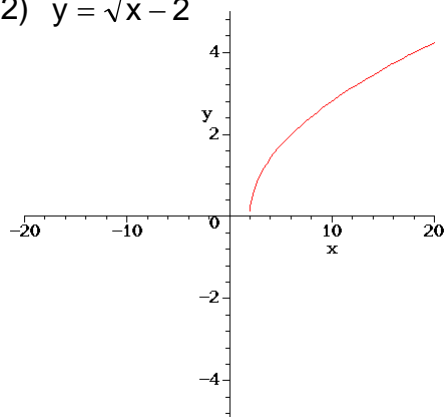R = All real numbers

10) $y = \dfrac{-x^3}{2} + 5$

D = All real numbers
R = All real numbers
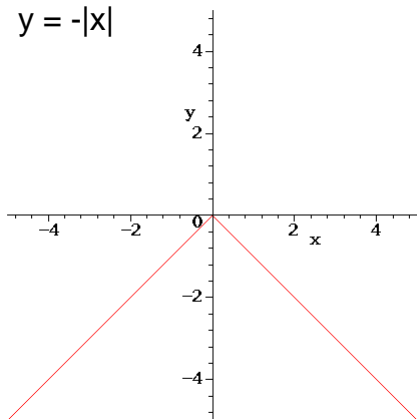
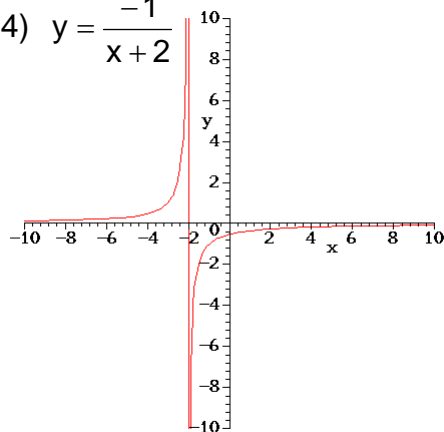11) $y = -\sqrt{x}$

D = x ≥ 0
R = y ≤ 0
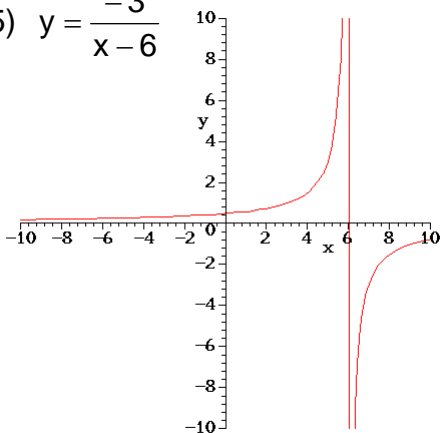
12) $y = \sqrt{x-2}$

D = x ≥ 1
R = y ≥ 0

13) y = -|x|

D = All real numbers
R = y ≤ 0

14) $y = \dfrac{-1}{x+2}$

D = All real numbers, but x = -2
R = All real numbers, but y = 0

15) $y = \dfrac{-3}{x-6}$

D = All real numbers, but x = 6
R = All real numbers, but y = 0