# Types of Mathematical Averages and Their Formulas

http://www.had2know.com/images/vo1.png

In mathematics, the average of two or more numbers is called the mean. The most commonly used mean is called the *arithmetic mean*, in which you add up all the values and divide by the number of values. For example, the arithmetic mean of 2, 5, and 14 is (2+5+14)/3 = 7. However, there is more than one kind of mean, including the geometric mean, harmonic mean, root mean square, and several others.  
  
The essential property of any mean is that it must fall between the highest value and the lowest value. When computing means, the type of mean you need to use depends on the type of data you are analyzing. Several formulas and examples are discussed below.

## Geometric Mean

The geometric mean of two numbers *x* and *y* is sqrt(*xy*). If you have three numbers *x*, *y*, and *z*, their geometric mean is cbrt(*xyz*), where "cbrt" means the cube root. For *n* numbers, the harmonic mean is  
  
(*x*1·*x*2·...*x*n)1/n  
  
Business analysts and scientists use the geometric mean to find the average growth rate of a process. For example, suppose a business's profits grow by 25% one year, and by 45.8% the next year. To find the average yearly percent growth rate, you must take the geometric mean of 1.25 and 1.458.  
  
sqrt[(1.25)(1.458)]  
= sqrt[1.8225]  
= 1.35  
  
Thus, the average growth rate over the two years was 35%. Compare this to the result you would get if you took the arithmetic mean of 25 and 45.8. Since (25+45.8)/2 = 35.4, you would get an answer that is too high.

## Harmonic Mean

In science in business applications, the harmonic mean is used to average ratios. For two numbers *x* and *y*, the harmonic mean is 2*xy*/(*x+y*). For three numbers *x*, *y*, and *z*, the harmonic mean is 3*xyz*/(*xy+xz+yz*). For *n* numbers, the harmonic mean is  
  
(1/*n*)(1/*x*1 + 1/*x*2 + ... + 1/*x*n)-1  
  
There are many instances where the harmonic mean is the most appropriate way to find the average value.  
  
For example, suppose a man drives at a speed of 80 k/h for 100 kilometers (1.25 hours), and then drives at a speed of 40 k/h for the next 100 km (2.5 hours). The average speed of the car for the entire 200 km trip is total distance divided by total time. Since 200/(1.25+2.5) = 53.33, the average speed is 53.33 k/h. This is equivalent to the harmonic mean of 80 and 40. Observe:  
  
2(80)(40)/(80+40)  
= 6400/120  
= 53.33

If we used the formula, we would have the same answer after we solve for the variable x  
200/x = 100/40+100/80   
  
In business, investors use the harmonic mean to compute the average price/earning ratio of a stock portfolio. For example, suppose you have three stocks, and their P/E ratios are 8, 18, and 30. The average P/E ratio of the three stocks is   
  
3(8)(18)(30)/(144+240+540)  
= 12960/924  
= 14.026

## Root Mean Square (Quadratic Mean)

The root mean square, aka quadratic mean, is used in many engineering and statistical applications, especially when there are data points that can be negative. The standard deviation of a set of numbers is an example of the root mean square. (It is the root mean square of the differences between each data point and the arithmetic mean.) If you have two numbers *x* and *y*, the quadratic mean is sqrt[(*x*2 + *y*2)/2]. For *n* variables, it is  
  
sqrt[(*x1*2 + *x2*2 + ... + *xn*2)/*n*]  
  
For example, suppose you have this set of numbers: -10, -5, -4, 1, 6, 7. The root mean square is  
  
sqrt[(100+25+16+1+36+49)/6]  
= sqrt(227/6)  
= 6.15  
  
which can be interpreted as the average positive value.

## Contraharmonic Mean

The contraharmonic mean of *x* and *y* is (*x*2 + *y*2)/(*x* + *y*). For *n* values, the contra- harmonic mean is  
  
(*x1*2 + *x2*2 + ... + *xn*2)/(*x1* + *x2* + ... + *xn*)  
  
For example, the contraharmonic mean of 1, 3, 5, and 7 is  
  
(1+9+25+49)/(1+3+5+7) = 84/16 = 5.25

## Other Means

[(*x*p + *y*p)/2]1/p    (Power Mean)  
  
[(*x*p - *y*p)/(p(*x* - *y*))]1/(p-1)    (Stolarsky Mean)  
  
        sqrt[(*x*2 + *xy* + *y*2)/3] when p = 3  
  
(*x*p + *y*p)/(*x*p-1 + *y*p-1)    (Lehmer Mean)  
  
[(*x*p + *y*p)/(*x*r + *y*r)]1/(p-r)  
  
[(r(*x*p - *y*p))/(p(*x*r - *y*r))]1/(p-r)  
  
[(*x*p*y*r + *x*r*y*p)/2]1/(p+r)  
  
(*x* - *y*)/(Ln(*x*) - Ln(*y*))    (Log Mean)  
  
(*x*Ln(*x*) + *y*Ln(*y*))/(Ln(*x*) + Ln(*y*))  
  
(*x* + sqrt(*xy*) + *y*)/3    (Heronian Mean)  
  
(1/e)(*xx*/*yy*)1/(x-y), e = 2.718281828....    (Identric Mean)  
  
(e)(*xy*/*yx*)1/(y-x)  
  
(*xxyy*)1/(x+y)  
  
(*xyyx*)1/(x+y)

## Mean Inequalities

Some means are in a constant relationship to one another. If we denote the arithmentic mean of *x* and *y* by **A**, their geometric mean by **G**, their harmonic mean by **H**, their root mean square by **R**, and their contraharmonic mean by **C**, then the following chain of inequalities is always true  
  
**C** ≥ **R** ≥ **A** ≥ **G** ≥ **H**

**Weighted Average**

A method of [computing](http://www.mathwords.com/c/compute.htm) a kind of [arithmetic mean](http://www.mathwords.com/a/arithmetic_mean.htm) of a [set](http://www.mathwords.com/s/set.htm) of numbers in which some [elements](http://www.mathwords.com/e/element_of_a_set.htm) of the set carry more importance (weight) than others.

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| Example: | Grades are often computed using a weighted average. Suppose that homework counts 10%, quizzes 20%, and tests 70%.  If Pat has a homework grade of 92, a quiz grade of 68, and a test grade of 81, then  Pat's overall grade = (0.10)(92) + (0.20)(68) + (0.70)(81)             = 79.5 |