Derivatives, Integrals, and Properties
Of Inverse Trigonometric Functions and Hyperbolic Functions

(On this handout, a represents a constant, u and x represent variable quantities)

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad (|u|<1)$$

$$\frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad (|u|<1)$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1}u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad (|u|>1)$$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad (|u|>1)$$

$$\frac{d}{dx}\cot^{-1}u = \frac{-1}{1+u^2}\frac{du}{dx}$$

Identities for Hyperbolic Functions

$$\sinh 2x = 2\sinh x\cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x = 1 - \operatorname{sech}^2 x$$

$$\coth^2 x = 1 + \operatorname{csch}^2 x$$

Integrals Involving Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad (\text{Valid for } u^2 < a^2)$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad (\text{Valid for all } u)$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad (\text{Valid for } u^2 > a^2)$$

Derivatives of Hyperbolic Functions

$$\frac{d}{dx}\sinh u = \cosh u\frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \sinh u\frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\coth u = -\operatorname{csch}^2 u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u\tanh u\frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u\coth u\frac{du}{dx}$$

The Six Basic Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Inverse Hyperbolic Identities

$$\operatorname{sech}^{-1} x = \cosh^{-1}\left(\frac{1}{x}\right)$$

$$\operatorname{csch}^{-1} x = \sinh^{-1}\left(\frac{1}{x}\right)$$

$$\coth^{-1} x = \tanh^{-1}\left(\frac{1}{x}\right)$$

Integrals of Hyperbolic Functions

$$\begin{aligned}
\int \sinh u \, du &= \cosh u + C \\
\int \cosh u \, du &= \sinh u + C \\
\int \operatorname{sech}^2 u \, du &= \tanh u + C \\
\int \operatorname{csch}^2 u \, du &= -\coth u + C \\
\int \operatorname{sech} u \tanh u \, du &= -\operatorname{sech} u + C \\
\int \operatorname{csch} u \coth u \, du &= -\operatorname{csch} u + C
\end{aligned}$$

Integrals Involving Inverse Hyperbolic Functions

$$\begin{aligned}
\int \frac{1}{\sqrt{a^2+u^2}} \, du &= \sinh^{-1}\left(\frac{u}{a}\right) + C && (a > 0) \\
\int \frac{1}{\sqrt{u^2-a^2}} \, du &= \cosh^{-1}\left(\frac{u}{a}\right) + C && (u > a > 0) \\
\int \frac{1}{a^2-u^2} \, du &= \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C & (\text{if } u^2 < a^2) \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C & (\text{if } u^2 > a^2) \end{cases} \\
\int \frac{1}{u\sqrt{a^2-u^2}} \, du &= -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C && (0 < u < a) \\
\int \frac{1}{u\sqrt{a^2+u^2}} \, du &= -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C
\end{aligned}$$

Derivatives of Inverse Hyperbolic Functions

$$\begin{aligned}
\frac{d}{dx}\sinh^{-1} u &= \frac{1}{\sqrt{1+u^2}}\frac{du}{dx} \\
\frac{d}{dx}\cosh^{-1} u &= \frac{1}{\sqrt{u^2-1}}\frac{du}{dx} && (u > 1) \\
\frac{d}{dx}\tanh^{-1} u &= \frac{1}{1-u^2}\frac{du}{dx} && (|u| < 1) \\
\frac{d}{dx}\operatorname{csch}^{-1} u &= \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx} && (u \neq 0) \\
\frac{d}{dx}\operatorname{sech}^{-1} u &= \frac{-1}{u\sqrt{1-u^2}}\frac{du}{dx} && (0 < u < 1) \\
\frac{d}{dx}\coth^{-1} u &= \frac{1}{1-u^2}\frac{du}{dx} && (|u| > 1)
\end{aligned}$$

Expressing Inverse Hyperbolic Functions As Natural Logarithms

$$\begin{aligned}
\sinh^{-1} x &= \ln(x + \sqrt{x^2+1}) && (-\infty < x < \infty) \\
\cosh^{-1} x &= \ln(x + \sqrt{x^2-1}) && (x \geq 1) \\
\tanh^{-1} x &= \frac{1}{2}\ln\frac{1+x}{1-x} && (|x| < 1) \\
\operatorname{sech}^{-1} x &= \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) && (0 < x \quad 1) \\
\operatorname{csch}^{-1} x &= \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right) && (x \neq 0) \\
\coth^{-1} x &= \frac{1}{2}\ln\frac{x+1}{x-1} && (|x| > 1)
\end{aligned}$$

Alternate Form For Integrals Involving Inverse Hyperbolic Functions

$$\begin{aligned}
\int \frac{1}{\sqrt{u^2 \pm a^2}} \, du &= \ln(u + \sqrt{u^2 \pm a^2}) + C \\
\int \frac{1}{a^2-u^2} \, du &= \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C \\
\int \frac{1}{u\sqrt{a^2 \pm u^2}} \, du &= -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2 \pm u^2}}{|u|}\right) + C
\end{aligned}$$